Fibonacci - Medina

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Introducción

Límites A,B,C,D

Teorema 1 (Límite A)**.** *Sea* $k, m, r \in \mathbb{N}$, $m \geqslant 1$, $r = 2^m$, $x \in \mathbb{R}$, $-1 < x < 1$, $x \neq \{\frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\phi_1(x) = \sum_{k=0}^{\infty} \varphi_{rk} x^{rk} = \frac{\varphi_r x^r}{x^{2r} - \beta_r x^r + 1} \tag{1}$$

Teorema 2 (Límite B)**.** *Sea* $k, m, r \in \mathbb{N}$, $m > 1$, $r = 2^m$, $x \in \mathbb{R}$, $-1 < x < 1$, $x \neq \{\frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\sum_{k=0}^{\infty} \varphi_{rk+\frac{r}{2}} x^{rk} = \frac{\varphi_{\frac{r}{2}}(x^r + 1)}{x^{2r} - \beta_r x^r + 1} \tag{2}$$

Corolario 1 (Límite C)**.** *Sea* $k, m, r \in \mathbb{N}$, $m \geqslant 1$, $r = 2^m$, $x \in \mathbb{R}$, $-1 < x < 1$

$$\sum_{k=0}^{\infty} (-1)^k \varphi_{rk} x^{rk} = \frac{-\varphi_r x^r}{x^{2r} + \beta_r x^r + 1} \tag{3}$$

Corolario 2 (Límite D)**.** *Sea* $k, m, r \in \mathbb{N}$, $m > 1$, $r = 2^m$, $x \in \mathbb{R}$, $-1 < x < 1$

$$\sum_{k=0}^{\infty} (-1)^k \varphi_{rk+\frac{r}{2}} x^{rk} = \frac{\varphi_{\frac{r}{2}}(1 - x^r)}{x^{2r} + \beta_r x^r + 1} \tag{4}$$

Usando variable compleja tenemos que:

$$z = x + iy \tag{5}$$

Y usando (1) obtenemos la siguiente representación:

Teorema 3. *Sea* $k, m, r \in \mathbb{N}$, $m \geqslant 1$, $r = 2^m$, $x \in \mathbb{R}$, $-1 < x < 1$, $x \neq \{\frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\phi_1(z) = \phi_1(x) + i\phi_1(y) = \sum_{k=0}^{\infty} \varphi_{rk} x^{rk} + i \sum_{k=0}^{\infty} \varphi_{rk} y^{rk} \tag{6}$$

Región separable

Sea $r \geqslant 4$, $\varphi(4) = 3$, $\phi, \tau, \alpha, b \in \mathbb{R}$ igual a

$$\begin{aligned} \alpha &= \frac{1}{\sqrt{5}} \\ \psi &= \frac{1+\sqrt{5}}{2} \quad \tau = \frac{1-\sqrt{5}}{2} \\ b &= \frac{\psi^4}{\log \psi} - \frac{\tau^4}{\log \tau} + \frac{3}{5} \end{aligned}$$

Sea $x, r \in \mathbb{R}$, $r \geqslant 4$, $j = (-1)^{\frac{2}{r}} \in \mathbb{C}$, $r' = \frac{r}{2}$.
Definimos β de la siguiente manera

$$\beta(r) = \psi^r + \tau^r = \Big(\frac{1+\sqrt{5}}{2}\Big)^r + \Big(\frac{1-\sqrt{5}}{2}\Big)^r$$

Definimos Θ y Υ

$$\begin{aligned} \Theta(x, r) &= \frac{\varphi_{r'}\Theta(x, r') + \varphi_{r'}\Theta(jx, r')}{\varphi_r} = \frac{x^r}{x^{2r} - \beta(r)x^r + 1} \\ \Upsilon(r) &= (2 - \beta(r))\Big(\frac{\psi^r}{\log \psi} - \frac{\tau^r}{\log \tau} - b\Big) \end{aligned}$$

Teorema 4. *Sea* $x, r \in \mathbb{R},\ r \geqslant 4,\ -1 \leqslant x \leqslant 1,\ x \neq \{0, \frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$
Sea Φ *igual a*

$$\Phi_3(x,r) = \alpha\Theta^2(x,r)\Big(\Upsilon(r)((\beta(r))' + \frac{\log x}{x^r} - x^r \log x) + (\Upsilon(r))'\frac{1}{\Theta(x,r)}\Big)$$

Entonces Φ *cumple con*

$$\int \Phi_3(x,r)dr = \alpha\Upsilon(r)\Theta(x,r) \tag{7}$$

Lema 1. *Sea* $x, r \in \mathbb{R},\ r \geqslant 4,\ -1 \leqslant x \leqslant 1,\ x \neq \{0, \frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\varphi_3(x,r) = \alpha\frac{\Theta^2(x,r)}{x^{2r}}\Big[\Upsilon(r)\Big((\beta(r))'x^r - \log x(x^{2r}-1)\Big) + (\Upsilon(r))'\Big(x^{2r} - \beta(r)x^r + 1\Big)\Big] \tag{8}$$

Lema 2. *Sea* $r \in \mathbb{R},\ r \geqslant 4$

$$\varphi_3(r) = \alpha\Big(\frac{(\Upsilon(r))'}{(2-\beta(r))} + \frac{(\beta(r))'\Upsilon(r)}{(2-\beta(r))^2}\Big) = \frac{(\psi^r - \tau^r)}{\sqrt{5}} \tag{9}$$

0.1 Integral A,B

Teorema 5 (Integral A)**.** *Sea* $x, r, b \in \mathbb{R},\ r \geqslant 4,\ -1 \leqslant x \leqslant 1,\ x \neq \{0, \frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\int \varphi(x,r)x^r dr = \alpha\Upsilon(r)\Theta(x,r) + b \tag{10}$$

Teorema 6 (Integral B)**.** *Sea* $x, r \in \mathbb{R},\ r \geqslant 4,\ -1 < x < 1,\ x \neq \{0, \frac{1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\}$

$$\int_4^\infty -\varphi(x,r)x^r dr = \alpha\Big(\frac{3x^4}{x^8 - 7x^4 + 1}\Big) \tag{11}$$